EXPLANATION OF THE INNSBRUCK DOUBLE DELAYED CHOICE EXPERIMENT

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The theory of elementary waves (TEW) accounts for the "double-delayed-choice" (DDC) experiment performed at Innsbruck[1], as well as other similar experiments, in a manner essentially the same as for single particle phenomena. Waves from the two polarizers impinge on the two-particle source where they stimulate the emission of particle pairs, each particle of which then follows its wave to its polarizer.[2]

At Innsbruck, `rotation' of a polarizer was accomplished through the use of a modulator to rotate the photon polarization along with a stationary polarizer. To simplify the discussion, in what follows I will refer to this modulator-polarizer combination as simply "the polarizer", and will speak simply of polarizer rotations. In addition, I will assume, as was assumed at Innsbruck, that the down-conversion source employed, including the various half-wave plates and other devices, does indeed produce photon pairs in an HV − VH state, and I will refer simply to the collection of devices as "the source" and will treat the waves incident on the two sides as interacting directly with one another at "the source".

Individual elementary waves have an inherent polarization. Along any particular line of propagation, individual waves of all polarizations are present. A polarizer divides the individual waves into two groups. If the polarizer is orientated with horizontal (H) at angle θ and vertical (V) at angle $(\theta + \pi/2)$, an individual elementary wave at angle ϕ will contribute to H a component proportional to $\cos(\theta - \phi)$ and to V a component proportional to $\sin(\theta - \phi)$.

The waves travelling from each polarizer to the source might be described either as a collection of individual elementary waves at all polarizations, or as two "polarized" waves (in the usual sense of "polarization") at right angles to one another, with the individual waves contributing to the two "polarized" waves as just indicated. The combination of two such "polarized" waves simply reduces to the overall collection of individual waves making them up. Due to the birefringence of the polarizing crystals employed, the H wave combination originates from a detector at one angle relative to the crystal and the V wave from a detector at a different angle. A particle photon following a wave from

[1] G. Weihs, T. Jennewein, C. Simon, H. Weinfurter, and A. Zeilinger, *Phys. Rev. Lett.* **81**, #23 (1998) 5039-5043.

[2] In earlier attempts to explain this phenomenon, I mistakenly thought that the explanation would involve rotation of the waves upon rotation of the polarizers, `jumping' of particles upon meeting the new waves, etc.—in short, new phenomena taking place away from the two-particle source. Such an approach, however, would not only be contradicted by Bell's theorem, it would be inconsistent with the essential ideas of TEW. The explanation offered here does not involve any such devices. The correct, experimentally verified result follows directly from wave interactions at the two-particle source.

source to crystal will thus deflect toward one detector or the other depending on which specific individual wave was being followed—depending, that is, on which wave collection, H or V, includes the wave being followed by the particle.

Rotation of a polarizer thus does not affect the waves travelling toward the source.[3] Whatever orientation a polarizer is placed in, and whether it is rotated into that position in a delayed manner or not, the H and V sides simply capture those particles following waves present in the collection they represent.

At the source, every individual wave on each side interacts continually with every individual wave on the other side. (Individual waves only act in concert in stimulating particle emission.) The HV − VH form means that the H component of a wave on one side interacts with the V component of a wave on the other side, and vice versa, but with the latter amplitude subtracting from the former. For incident waves at angles ϕ_1 and ϕ_2 on sides 1 and 2 respectively, quantitatively the interaction is thus proportional to:

$$\cos(\theta - \phi_1)\sin(\theta - \phi_2) - \sin(\theta - \phi_1)\cos(\theta - \phi_2) = \sin(\phi_1 - \phi_2), \tag{1}$$

where θ is the H orientation of the source.

Consider the collection of waves making up polarizer orientation H at angle θ_1 on side 1 and H at angle θ_2 on side 2. These waves interact at the source with an amplitude proportional to:

$$\tfrac{1}{4}\int d\phi_1 \int d\phi_2 \cos(\theta_1 - \phi_1)\sin(\phi_1 - \phi_2)\cos(\theta_2 - \phi_2)$$

$$= \tfrac{1}{2}\int d\phi_2 \sin(\theta_1 - \phi_2)\cos(\theta_2 - \phi_2) = \sin(\theta_1 - \theta_2), \tag{2}$$

where the integrals are performed over all angles along with a factor of ½ instead of between $\pm\pi/2$ on either side of ϕ_1 and ϕ_2. The cosine factors give the amplitude of the contribution of each individual elementary wave. The square of amplitude (2) then gives the probability that a particle pair is emitted in such a manner as to follow some portion of the H wave on both sides and hence to be observed as H at both polarizers. The particles, again, simply follow their wave to the polarizer.

It is not necessary that the two polarizers actually be in this orientation at the moment the pair is emitted. The individual waves interact at the source in the same manner regardless of the polarizer orientations. Stimulation at angles θ_1 and θ_2 is taking place at all times regardless of the orientations, as is stimulation at all other combinations of orientations.

Because the interaction that produces the sin squared result occurs at the particle source, where the waves from the two sides interact directly with one

[3] I apologize for my statements to the contrary in previous treatments of this experiment.

another (locally), this explanation does not contradict Bell's theorem.[4] (Bell applied his analysis to pairs of spin ½ fermions, not photons. However, a modification of his analysis leads to similar conclusions in the case of photon pairs.) Indeed, equation (2) above, squared to obtain a probability of emission, is not of the form of Bell's equation (2),[5] refuted by his analysis. Mathematically, Bell's analysis—that is, the spin 1 modification of that analysis—does not apply.

Bell's approach to resolving the "weirdness" of quantum mechanics, modified to apply to photons as just indicated, was to ask if one can devise a viable theory in which each photon has a single polarization, with a value determined by some additional variables. The answer was no. But TEW doesn't attempt to explain things in this manner. As with other phenomena, the explanation involves multiple waves with multiple polarizations (analogous to multiple paths in the double slit experiment and other phenomena treated previously), which waves interfere with one another in stimulating pair emission. The particles have only one state, but the waves exist in many, with interference taking place between the waves prior to emission of the particles. It is this physical difference in approach that accounts for the difference between the equations (2) in the two analyses.

Because wave interference determines the emission probability at any combination of orientations, the probabilities of emission at various different orientations will not relate to one another in a classical manner. This is better illustrated if one derives the sin squared relationship using the usual two "polarized" waves at right angles to one another on each side instead of the individual elementary waves polarized at all angles. As indicated above, the two descriptions are equivalent. So suppose "polarized" waves at angles θ_1 and $\theta_1 + \pi/2$ exist initially on side 1 and similarly at angles θ_2 and $\theta_2 + \pi/2$ on side 2. Each wave on side 1 interacts with each wave on side 2 with the $\sin(\theta_1 - \theta_2)$ factor. (The derivation of this relationship is exactly that which led to equation (2)). If one now rotates θ_1 to a new angle θ_1' and θ_2 to θ_2', each new incoming wave at the new angle is equivalent to a combination of the original two waves on that side, with a cos factor. What we are interested in is the manner in which these wave combinations were interacting at the source prior to the rotations; it is only particles emitted in response to waves that make up a portion of these wave combinations that will be observed at the new angles. The amplitude for observation at θ_1' and θ_2' is thus:

$$\cos(\theta_1' - \theta_1)\sin(\theta_2 - \theta_1)\cos(\theta_2' - \theta_2) \qquad \text{(HH)}$$

$$+ \cos(\theta_1' - \theta_1)\sin(\theta_2 + \pi/2 - \theta_1)\cos(\theta_2' - \theta_2 - \pi/2) \qquad \text{(HV)}$$

$$+ \cos(\theta_1' - \theta_1 - \pi/2)\sin(\theta_2 - \theta_1 - \pi/2)\cos(\theta_2' - \theta_2) \qquad \text{(VH)}$$

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[4] J. S. Bell, "On the Einstein-Podolsky-Rosen paradox", *Physics* **1** (1964) 195-200.
[5] Ref. 4.

$$+\ \cos(\theta_1' - \theta_1 - \pi/2)\sin(\theta_2 + \pi/2 - \theta_1 - \pi/2)\cos(\theta_2' - \theta_2 - \pi/2) \qquad (VV)$$

$$= \cos(\theta_1' - \theta_1)\sin(\theta_2 - \theta_1)\cos(\theta_2' - \theta_2)$$

$$+\ \cos(\theta_1' - \theta_1)\cos(\theta_2 - \theta_1)\sin(\theta_2' - \theta_2)$$

$$-\ \sin(\theta_1' - \theta_1)\cos(\theta_2 - \theta_1)\cos(\theta_2' - \theta_2)$$

$$+\ \sin(\theta_1' - \theta_1)\sin(\theta_2 - \theta_1)\sin(\theta_2' - \theta_2)$$

$$= \cos(\theta_1' - \theta_1)[\sin(\theta_2 - \theta_1)\cos(\theta_2' - \theta_2) + \cos(\theta_2 - \theta_1)\sin(\theta_2' - \theta_2)]$$

$$-\ \sin(\theta_1' - \theta_1)[\cos(\theta_2 - \theta_1)\cos(\theta_2' - \theta_2) - \sin(\theta_2 - \theta_1)\sin(\theta_2' - \theta_2)]$$

$$(3) \qquad = \cos(\theta_1' - \theta_1)\sin(\theta_2' - \theta_1) - \sin(\theta_1' - \theta_1)\cos(\theta_2' - \theta_1) = \sin(\theta_2' - \theta_1').$$

As before, the result in equation (3) is the same regardless of the initial orientations. The use of delayed rotations changes nothing. Only the orientation of the polarizers at the moment when the particles arrive will affect the outcome. It is not the case that each particle is emitted in response to one of the two "polarizations" on each side, followed by a `jump' to a new wave upon rotation. The waves at *both* of the two initial perpendicular "polarizations" on each side participate in determining the outcome following rotations.

The derivation of equation (3) makes clear that, when one rotates a polarizer, the change to the probability is determined by interference between the four initial waves, two on each side. When one rotates, it is the *wave amplitudes* that combine to determine the new amplitude—and hence the new probability—not the probabilities themselves. This, again, is why the classical relationships between the probabilities at various orientations do not obtain.

At Innsbruck, the polarizer on one side was rotated between angles 0° and 45° and on the other side between 22.5° and 67.5°. With the three combinations 0° and 22.5°, 45° and 22.5°, and 45° and 67.5°, the probability of coincidences is proportional to $\sin^2 22.5° = 0.1464$, while with the combination 0° and 67.5° the probability is proportional to $\sin^2 67.5° = 0.8536$. It is alleged that these probabilities are incompatible with one another, that there is no combination of H and V observations on the two sides that would reproduce all four probabilities—that is, unless there is interaction between the two sides. But in TEW the interaction of relevance does involve both sides, as captured by equation (2) or (3). Furthermore, the single expression given in equation (2) captures all four probabilities simultaneously—that is, without there being any question of polarizer rotations. Nothing changes upon rotation of a polarizer. There is no non-local interaction to consider because nothing changes. The polarizers simply passively observe the particles that arrive. And, again, the fact that the four probabilities do not add up as one might expect classically is due to the fact of wave interference. The amplitudes do `add up' correctly, as shown by the derivations of equations (2) and (3).

TEW does, indeed, account in a local manner for all phenomena described by quantum mechanics. We do live in a local, deterministic universe.